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# The acoustics of turbulence near sound-absorbent liners

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Acoustic liners are often perforated screens backed by sound-absorbent material. Turbulence can interact with these screens to generate additional sound. The dynamics of the generation process is examined in this paper, where the liner is modelled as an infinite rigid plane boundary with a homogeneous array of circular orifices or rigid pistons. The acoustic properties of these boundaries are derived in the long wavelength limit. Small-scale turbulence is scattered by individual apertures into sound. Acoustically transparent surfaces support dipole scattering centres while more 'opaque' surfaces have monopoles at the apertures which convert turbulence into sound more effectively. It is shown that the process can be described once the response of an individual aperture in an infinite baffle is known. At low Mach numbers the screen can increase the sound radiated by adjacent turbulence by a factor equal to the inverse fourth power of the Mach number. Mean-flow effects are ignored but they are thought to increase the effects deduced in this preliminary study.

## 1. Introduction

Sound propagating within a vessel or duct is known to be very effectively absorbed when the vessel walls are treated with sound-absorbent material. This principle is finding increasing application in various schemes to control the noise of aircraft engines. There the noise often coexists with turbulence, the most common situation being one in which sound propagates along a duct containing a moving stream that is bounded by a turbulent boundary layer. It is known (Mechel 1960; Mechel, Mertens & Schilz 1962) that additional sound can be generated by linings in these circumstances so that practical schemes must maximize sound absorption simultaneously with a control of the new source mechanisms. This optimization is difficult while the basic mechanisms remain unidentified and experimental data restricted to a rather narrow range of parametric variation. This paper goes some way to filling this gap by demonstrating how turbulence can interact with small-scale features of an acoustic liner to generate sound rather effectively. The analysis involves a more precise treatment of liner properties than is usual since the essence of the scattering process is in the small-scale features that are 'smeared over' in attributing an effective surface impedance to the liner. Precise descriptions are impractical for the liners in commercial use, and all except the most simplified geometrical arrangements are likely to provide intractable analytic problems.

Two liner geometries of the 'perforated screen' type are treated in this paper. They are chosen primarily because they are amenable to precise analysis but also because they probably model very closely the turbulence–liner interaction processes of practical interest. The first model is one in which an infinite plane thin rigid screen (at $z = 0$) is perforated with a homogeneous distribution of identical circular apertures. This screen is irradiated from one side ($z \geqslant 0$) by sound and turbulence, and the problem is to determine how much sound is finally radiated into the half space $z > 0$, that sound being composed of two parts: (*a*) the reflexion (minus absorption) of the incident sound and (*b*) the newly created sound that is scattered from the turbulence by the screen. To the upper half space this screen appears as an absorbent liner, the sound radiated through the screen representing the absorption. The second problem concerns an identical geometry but this time the apertures contain plane circular pistons whose motion is related to the piston force by a known impedance. Again the sound radiated to the upper half space is determined as a function of the incident sound field, local turbulence and screen properties.

Quite apart from the practical motivation of this problem there is a distinct theoretical interest in determining the criteria controlling the several mechanisms of sound generation by turbulence near an absorptive screen. It is known that if the surface is so 'fine-grained' that it even appears homogeneous and continuous on the smallest scale in the turbulent motion, then interaction of the flow with the continuous boundary could not materially influence the turbulence radiation efficiency (Powell 1960; Ffowcs Williams 1965). On the other hand this eventuality is unlikely since it requires that the flow Reynolds number based on aperture diameter be held smaller than unity, and if this is done the viscous layer, of thickness at least equal to the hole radius divided by the root of the product of the Reynolds and Strouhal numbers, will be so thick as to fill the hole and impede the motion. That would tend to restore the rigid surface condition and destroy the absorption properties. The degree to which this happens would, however, have to be subject to experimental checks. Conversion of turbulent energy into sound therefore rests on small-scale details of the liner surface. The fibrous surfaces have a confused detailed geometry which defies deterministic analysis, so that model problems have to be based on the so-called 'resonant cavity' type of surface, even though the resonant behaviour may be missing. These surfaces are perforated screens backed by some dissipative layer, which we model here in the very specific way that is already described.

Consider specifically the possibilities of sound production at the perforated screen irradiated by turbulence. There is an immediate dilemma. Should attention be concentrated on the apertures through which the turbulence will be driving an unsteady monopole producing mass flow, or should one concentrate on the physical rigid sections of the boundaries which can only support fluctuations of force that induce a dipole scattered field (Curle 1955)? Crudely speaking, since sound is scattered by the inhomogeneity one would emphasize the more obviously singular regions. For a sparsely perforated screen, each aperture is an obvious singularity and one would expect an efficient monopole radiation from the environment of each aperture. But when the holes are very wide, so that there is more aperture than screen as it were, then the rigid sections are the obviously singular regions and a dipole scattered field would be expected. This viewpoint is essentially correct as the following analysis will show, but the scale on which the parameters are measured to determine degree of monopole and dipole behaviour must be derived from the details of that analysis and cannot be convincingly argued *a priori*. It transpires that even small holes appear large at sufficiently low frequencies, for then the leakage through the screen in a period of oscillation can be very substantial even though the *rate* of leakage through a hole is small.

It is already clear from the mere existence of these scattering source terms that sound can be generated rather efficiently by turbulence interacting with the screen. In fact the efficiency with which turbulence energy is converted into sound can be increased by a factor equal to the fourth power of the ratio of sound speed to the root-mean-square turbulence velocity level. At low mean-flow Mach numbers ($M < 1$) this is a very substantial effect.

The direct effect of the mean flow on radiation is ignored in this analysis, though no suggestion is made that these effects are small. In fact the contrary is true, for it is anticipated that mean flow over a compliant boundary will have instabilities of the Kelvin–Helmholtz type (Benjamin 1963), which can obviously have dramatic effects on the acoustic properties of the surface layer (Miles 1956). These effects, however, enhance those considered here.

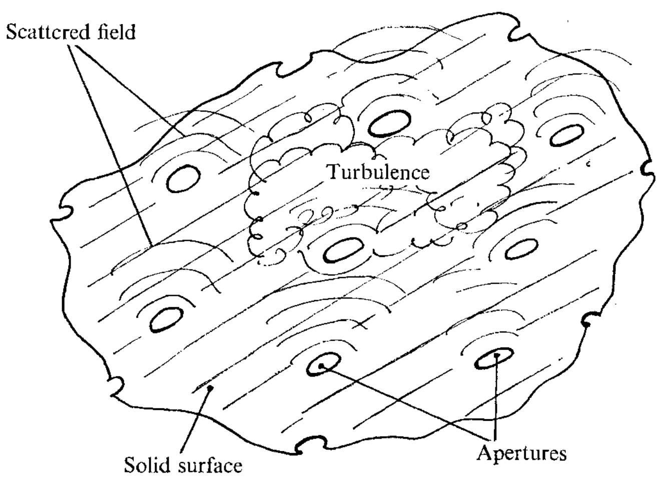


FIGURE 1. Diagrammatic view of turbulence inducing a scattered radiation field from a perforated boundary. The holes are circular and all of radius $a$, there being $N$ holes per unit area of surface.

## 2. The acoustic properties of turbulence near a perforated rigid screen

The model problem is illustrated in figures 1 and 2 together with the co-ordinate system. We seek a description of the sound radiated to large distances on the turbulent side of the screen ($z > 0$) at an angle $\theta$ with the surface normal. The source region will be in the neighbourhood of the co-ordinate origin. The screen is vanishingly thin and remains motionless under turbulent irradiation. It is perforated with a homogeneous distribution of identical circular apertures of radius $a$, densely packed on the 'acoustic wavelength' scale so that an acoustic wave would be reflected from the screen as if it were some continuous homogeneous boundary. The screen is surrounded on one side ($z > 0$) by locally turbulent fluid that asymptotes to an acoustic medium at rest with wave speed $c$ at infinity, while the fluid is an ideal acoustic medium everywhere on the opposite side.

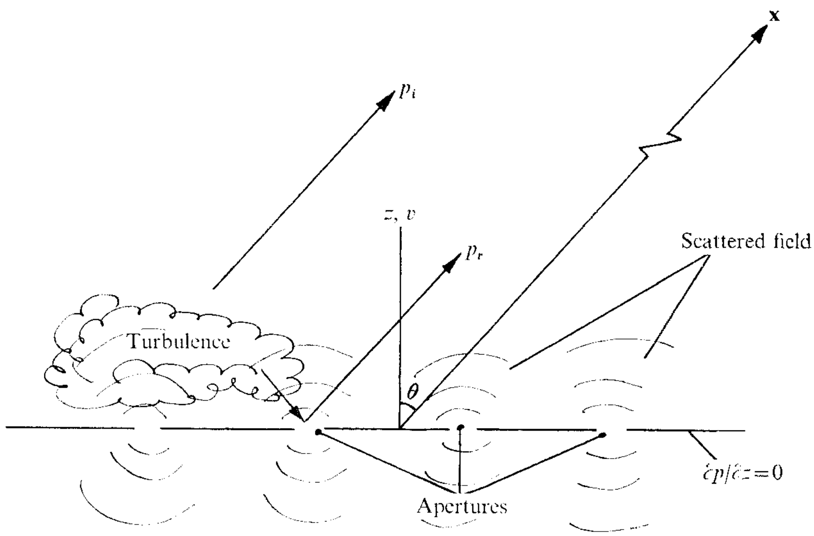


FIGURE 2. Diagram of the model problem with the co-ordinate systems.

According to Lighthill's (1952) acoustic analogy the field generated by turbulent flow in the vicinity of the perforated screen is given exactly as the solution to a quadrupole-driven wave equation subject to the boundary conditions of no velocity on the rigid sections of the screen and an outgoing wave condition at infinity. We consider here a low Mach number flow where the source and sound fields exist on clearly separated length scales. The source of scale $l$ generates sound at scale $l/M$. We will suppose that the scale $l$ is not large in comparison with the typical distance $(\pi N)^{-\frac{1}{2}}$ between perforations, $N$ being the number area density of the circular apertures. On the other hand $M$ is sufficiently small that there are many holes per acoustic wavelength, so that $lM^{-1}(\pi N)^{\frac{1}{2}} \gg 1$.

The analysis will be conducted for the particular frequency $\omega$. In the flow field this frequency is associated with a length scale $2\pi u'/\omega$, $u'$ being the root-mean-square turbulence level, and a radiation length scale $2\pi c/\omega$. The radiated wavenumber $\kappa = \omega/c$ is small enough for the following inequality to hold:

$$\kappa a < \kappa(\pi N)^{-\frac{1}{2}} \ll 1 \tag{1}$$

or, equivalently,

$$a \ll (\pi N)^{-\frac{1}{2}} \ll lc/u' = lM^{-1}. \tag{2}$$

Lighthill's (1952) analogy then allows us to determine the radiated sound pressure $p(\mathbf{x})$ at the distant field point $\mathbf{x}$ by solving the inhomogeneous Helmholtz equation:

$$(\nabla^2+\kappa^2)p = T \tag{3}$$

subject to an outgoing wave condition at infinity and $\partial p/\partial z = 0$ on the rigid sections of the screen. $T$ is written symbolically for the turbulence-generated inhomogeneity.

The required solution to this equation can be written down explicitly in terms of the Green's function for an infinite rigid boundary and the pressure gradient on the circular apertures $S_n$.

$$p(\mathbf{x}) = p_i(\mathbf{x})+p_r(\mathbf{x})-\frac{1}{2\pi}\sum_n\int_{S_n}\frac{\exp[i\kappa|\mathbf{x}-\mathbf{y}|]}{|\mathbf{x}-\mathbf{y}|}\frac{\partial p(\mathbf{y})}{\partial z}d^2\mathbf{y} \quad (z \geqslant 0), \tag{4}$$

i.e.
$$p(\mathbf{x}) = p_i(\mathbf{x})+p_r(\mathbf{x})-\frac{1}{2\pi}\sum_n\frac{\exp[i\kappa|\mathbf{x}-\mathbf{y}_n|]}{|\mathbf{x}-\mathbf{y}_n|}Q(\mathbf{y}_n) \quad (z \geqslant 0), \tag{5}$$

where
$$Q(\mathbf{y}_n) = \int_{S_n}\frac{\partial p(\mathbf{y})}{\partial z}d^2\mathbf{y}, \tag{6}$$

and $\mathbf{y}_n$ is the centre of the $n$th aperture.

Asymptotically, as $|\mathbf{x}|\to\infty$, the radiation field of a supposedly finite source region becomes a centred spherical wave. That wave is given by the asymptotic form of (5) as

$$p(\mathbf{x}) \sim p_i(\mathbf{x})+p_r(\mathbf{x}) \\ -\frac{\exp[i\kappa|\mathbf{x}-\mathbf{y}_0|]}{2\pi|\mathbf{x}|}\sum_n \exp[-i\kappa(y_{n1}-y_{01})\sin\theta]\,Q(\mathbf{y}_n) \quad \text{as} \quad |\mathbf{x}|\to\infty. \tag{7}$$

$\mathbf{y}_0$ is the centre of a reference orifice near the co-ordinate origin and the suffix 1 implies the co-ordinate in the screen surface along which the ray is moving towards the field point $\mathbf{x}$ at an angular elevation $\theta$ relative to the surface normal. The time dependence is taken to be $e^{-i\omega t}$ throughout. $p_i(\mathbf{x})$ is the pressure that would be radiated from the source distribution $T$ if it were unbounded by the perforated screen and $p_r(\mathbf{x})$ is the reflexion of this sound in a uniform rigid screen so that $p_i(x)+p_r(\mathbf{x})$ is known trivially as the field of turbulence near the specularly reflecting solid surface at $z = 0$ (Powell 1960).

$$p_r(\mathbf{x}) \sim \frac{-i\exp[i\kappa|\mathbf{x}-\mathbf{y}_0|]}{2\pi|\mathbf{x}|}\kappa\cos\theta \\ \times\int_{S(z=0)}p_i(\mathbf{y})\exp[-i\kappa(y_1-y_{01})\sin\theta]\,d^2\mathbf{y} \quad \text{as} \quad |\mathbf{x}|\to\infty. \tag{8}$$

Within an aperture in the screen surface $z = 0$ the pressure $p_i = p_r$ is undisturbed from that in an unbounded field forced by the source distribution $T$. Equation (4) is then an integral equation from which $\partial p/\partial z$ is to be determined.

$$p_i(\mathbf{y}) = \frac{1}{2\pi}\sum_j\int_{S_j}\frac{\exp[i\kappa|\boldsymbol{\xi}-\mathbf{y}|]}{|\boldsymbol{\xi}-\mathbf{y}|}\frac{\partial p(\boldsymbol{\xi})}{\partial z}d^2\boldsymbol{\xi}, \quad \mathbf{y}\in S_j. \tag{9}$$

We now concentrate on a particular aperture $S_n$ and rewrite (9):

$$p_i(\mathbf{y})-\frac{1}{2\pi}\sum_{j\neq n}\int_{S_j}\frac{\exp[i\kappa|\boldsymbol{\xi}-\mathbf{y}|]}{|\boldsymbol{\xi}-\mathbf{y}|}\frac{\partial p(\boldsymbol{\xi})}{\partial z}d^2\boldsymbol{\xi} = \frac{1}{2\pi}\int_{S_n}\frac{1}{|\boldsymbol{\xi}-\mathbf{y}|}\frac{\partial p(\boldsymbol{\xi})}{\partial z}d^2\boldsymbol{\xi},\ y\in S_n. \tag{10}$$

The neglect of phase variation on the right-hand side of this equation is permitted since $\kappa a \ll 1$, and this step reduces the question of determining $\partial p/\partial z$ to Copson's (1947) problem of determining the charge distribution required on a disk to maintain a specified potential. We neglect the effect of small variations in pressure within an individual aperture so that the 'mass flux' through the aperture $S_n$ is given by Copson's theorem as

$$Q(\mathbf{y}_n) = 4ap_i(\mathbf{y}_n)-\frac{2a}{\pi}\sum_{j\neq n}\int_{S_j}\frac{\exp[i\kappa|\mathbf{y}_n-\boldsymbol{\xi}|]}{|\mathbf{y}_n-\boldsymbol{\xi}|}\frac{\partial p(\boldsymbol{\xi})}{\partial z}d\boldsymbol{\xi}. \tag{11}$$

Since, in our problem, the distance between holes is assumed to be much greater than the hole dimension, i.e. $a \ll (\pi N)^{-\frac{1}{2}}$, $|\mathbf{y}_n-\boldsymbol{\xi}|$ can be set equal to $|\mathbf{y}_n-\boldsymbol{\xi}_j|$, $\mathbf{y}_n$ being the centre of the $n$th aperture and $\boldsymbol{\xi}_j$ the centre of the $j$th aperture. Equation (7) then assumes a simpler form:

$$Q(\mathbf{y}_n) = 4ap_i(\mathbf{y}_n)-\frac{2a}{\pi}\sum_{j\neq n}\frac{\exp[i\kappa|\mathbf{y}_n-\boldsymbol{\xi}_j|]}{|\mathbf{y}_n-\boldsymbol{\xi}_j|}Q(\boldsymbol{\xi}_j). \tag{12}$$

From this we can estimate the scattered field described by the summation in equation (7).

$$\sum_n \exp[-i\kappa(y_{n1}-y_{01})\sin\theta]\,Q(\mathbf{y}_n) = 4a\sum_n \exp[-i\kappa(y_{n1}-y_{01})]\,p_i(\mathbf{y}_n) - \frac{2a}{\pi}\sum_n\sum_{j\neq n}\exp[i\kappa|\mathbf{y}_n-\boldsymbol{\xi}_j|-i\kappa(y_{n1}-y_{01})\sin\theta]\frac{Q(\boldsymbol{\xi}_j)}{|\mathbf{y}_n-\boldsymbol{\xi}_j|}. \tag{13}$$

The summation over $n$ in the last term of this equation can be performed by integration since the function to be summed is a smooth slowly varying function of $\mathbf{y}_n$ everywhere excepting the neighbourhood of the $j$th hole. However, the contribution to the integral from the region surrounding that hole is smaller by a factor $a(\pi N)^{\frac{1}{2}}$ than the other terms in the equation and can be neglected, since this ratio is small. Accordingly

$$\frac{2a}{\pi}\sum_n\sum_{j\neq n}\exp[i\kappa|\mathbf{y}_n-\boldsymbol{\xi}_j|-i\kappa(y_{n1}-y_{01})\sin\theta]\frac{Q(\boldsymbol{\xi}_j)}{|\mathbf{y}_n-\boldsymbol{\xi}_j|}$$

$$= \frac{2aN}{\pi}\int_0^\infty\int_0^{2\pi}\sum_i Q(\boldsymbol{\xi}_j)\exp[-i\kappa(\xi_{j1}-y_{01})\sin\theta+i\alpha\kappa(1-\sin\theta\cos\phi)]\,d\phi\,d\alpha \tag{14}$$

$$= \frac{4iaN}{\kappa\cos\theta}\sum_n Q(y_n)\exp[-i\kappa(y_{n1}-y_{01})\sin\theta]. \tag{15}$$

Equations (13) and (15) then combine to give

$$\sum_n \exp[-i\kappa(y_{n1}-y_{01})\sin\theta]\,Q(\mathbf{y}_n) = \frac{4a}{\{1+(4iaN/\kappa\cos\theta)\}}\sum_n p_i(\mathbf{y}_n)\exp[-i\kappa(y_{n1}-y_{01})\sin\theta]. \tag{16}$$

We now introduce into the equations a specification of the property that at low Mach numbers the acoustic and hydrodynamic fields exist on clearly separated length scales. The acoustic field is slowly varying on the scale of the perforations in the screen so that the sum can be evaluated by integration. The hydrodynamic part is not slowly varying in general so that no simplification of the summation is possible for this component.

We now decompose the pressure $p_i$ that would be incident on the position of the screen, $z = 0$, if the wave equation inhomogeneity were maintained at $T$ but the screen discarded, into a slowly varying acoustic part $p_a$ and a rapidly varying hydrodynamic part $p_h$.

$$p_i(\mathbf{y}_n) = p_a(\mathbf{y}_n) + p_h(\mathbf{y}_n), \tag{17}$$

$$\begin{aligned}\sum_n p_i(\mathbf{y}_n) \exp[-i\kappa(y_{n1}-y_{01})\sin\theta] &= N\int_{S(z=0)} p_a(\mathbf{y}) \exp[-i\kappa(y_1-y_{01})\sin\theta]\, d^2\mathbf{y} \\ &\qquad + \sum_n p_h(\mathbf{y}_n) \exp[-i\kappa(y_{n1}-y_{01})\sin\theta] \qquad (18)\\ &= N\int_{S(z=0)} p_i(\mathbf{y}) \exp[-i\kappa(y_1-y_{01})\sin\theta]\, d^2\mathbf{y} \\ &\qquad + \sum_n p_h(\mathbf{y}_n) \exp[-i\kappa(y_{n1}-y_{01})\sin\theta]. \qquad (19)\end{aligned}$$

This last step is permitted since by definition there is no element of $p_h$ that exists on an 'acoustic' scale, and the surface integral is the Fourier transform operation that selects the 'acoustic' part of the surface pressure field.

The integral in (19) can be recognized as a term proportional to the pressure $p_r$ reflected from a homogeneous rigid boundary at $z = 0$. Use of (8) allows (18) to be re-expressed as

$$\begin{aligned}\sum_n p_i(\mathbf{y}_n) \exp[-i\kappa(y_{n1}-y_{01})\sin\theta] &= \frac{2\pi N i|\mathbf{x}|}{\kappa\cos\theta} \exp[-i\kappa|\mathbf{x}-\mathbf{y}_0|]\, p_r(\mathbf{x}) \\ &\quad + \sum_n p_h(\mathbf{y}_n) \exp[-i\kappa(y_{n1}-y_{01})\sin\theta] \quad \text{as} \quad |\mathbf{x}|\to\infty. \qquad (20)\end{aligned}$$

Finally, by using this with (16), equation (7) becomes

$$\begin{aligned}p(\mathbf{x}) \sim p_i(\mathbf{x}) &+ \left\{1 + \frac{4aiN}{\kappa\cos\theta}\right\}^{-1} p_r(\mathbf{x}) \\ &- \frac{4a}{\{1 + 4aiN/\kappa\cos\theta\}} \sum_n p_h(\mathbf{y}_n) \frac{\exp(i\kappa|\mathbf{x}-\mathbf{y}_n|)}{2\pi|\mathbf{x}|} \quad \text{as} \quad |\mathbf{x}|\to\infty. \qquad (21)\end{aligned}$$

Evidently the perforated screen acts like a homogeneous boundary surface to acoustic waves with reflexion coefficient $R$ given by the form taken by (21) in the absence of a hydrodynamic field.

$$p(\mathbf{x}) = p_i(\mathbf{x}) + Rp_r(\mathbf{x}), \tag{22}$$

$$R = \{1 + (4aiN/\kappa\cos\theta)\}^{-1}. \tag{23}$$

In general, then,

$$p(\mathbf{x}) \sim p_i(\mathbf{x}) + Rp_r(\mathbf{x}) - 2R \sum_n 4ap_h(\mathbf{y}_n) \frac{\exp[i\kappa|\mathbf{x}-\mathbf{y}_n|]}{4\pi|\mathbf{x}-\mathbf{y}_n|} \quad \text{as} \quad |\mathbf{x}| \to \infty, \tag{24}$$

$$p(\mathbf{x}) \sim p_i(\mathbf{x}) + Rp_r(\mathbf{x}) + 2R \sum_n -Q_h(\mathbf{y}_n) \frac{\exp[i\kappa|\mathbf{x}-\mathbf{y}_n|]}{4\pi|\mathbf{x}-\mathbf{y}_n|} \quad \text{as} \quad |\mathbf{x}| \to \infty. \tag{25}$$

The second of these forms is given since it is suggestive of a physical interpretation of this final result.

$-Q_h(\mathbf{y}_n)$ is the monopole strength associated with mass flow through an infinitely baffled circular aperture centred at $\mathbf{y}_n$ that is driven by an incident incompressible field $p_h(\mathbf{y})$. This monopole is 'backed' by the screen, which has reflexion coefficient $R$ to the acoustic wave. The monopole on the near-side of the screen therefore radiates a field $(-e^{ikr}/4\pi r)\,Q(1+R)$. On the other side of the screen the mass flowing through the aperture induces a monopole of strength $+Q$ which radiates through the screen, with transmission coefficient $(1-R)$, a field $(e^{ikr}/4\pi r)\,Q(1-R)$. The resulting field from each hole is consequently

$$-2RQ(e^{ikr}/4\pi r),$$

and this is the form of the scattered field according to (25).

This suggests that the details of the equivalent monopoles at the orifices might be calculated according to incompressible flow theory, ignoring any interaction of adjacent apertures. The radiation properties of these monopoles should then be calculated as if the porous screen were a homogeneous semi-transparent surface with reflexion coefficient given by (23), a value that implies a normal surface impedance $Z$ equal to

$$Z = \frac{\rho c}{\cos\theta}\left\{\frac{1+R}{1-R}\right\} = \frac{\rho c}{\cos\theta} - i\rho c\,\frac{\kappa}{2aN}. \tag{26}$$

The real part of the surface impedance is constant at $\rho c/\cos\theta$, while the imaginary part is frequency- and geometry-dependent. Low frequency waves pass through the screen very easily with very little change, the effect of the screen being reduced if the apertures are either very numerous, $N$ large, or of large radius. When the angle $\theta$ is close to $\frac{1}{2}\pi$ and the sound ray is grazing the screen, then the solid portions become 'streamlines' and cannot scatter the wave field. Then, as we see, the impedance is that of infinite fluid, $\rho c/\cos\theta$, and the reflexion coefficient $R$ goes to zero.

The reflexion coefficient and impedance given in (23) and (26) respectively are fully consistent with those found for the same model problem by Hughes (1970) and Leppington & Levine (1971). Hughes obtained his solutions by an iteration scheme restricted to small hole dimensions and non-grazing incidence angles. Leppington & Levine derive the precise asymptotic form for a screen with regularly arranged apertures. Their solution confirms that (23) and (26) give the leading terms in a low frequency expansion which is uniformly valid for all incidence angles.

Limiting forms of the dependence of the sound field on the basic parameters are easily given. When the effective porosity is small, i.e.

$$4aN/\kappa\cos\theta \ll 1, \tag{27}$$

the quadrupole direct and reflected fields are supplemented by monopoles positioned at each aperture and radiating a scattered distant field of amplitude

$$\sum_n \frac{2a}{\pi|\mathbf{x}|} p_h(\mathbf{y}_n). \tag{28}$$

This field is independent of the radiation angle and of the number of holes per unit screen area, provided of course that the inequality (27) is met. The aperture-scattered field would then increase in direct proportion to the near-field pressure, giving a far-field intensity proportional to

$$\rho U^3 M a^2/|\mathbf{x}|^2. \tag{29}$$

If the opposite inequality to (27) applies, i.e.

$$4aN/\kappa \cos\theta \gg 1, \tag{30}$$

then the screen is relatively acoustically transparent because either the screen is set as a near 'streamline' or it is of high porosity.

The surface-scattered field is then that due to a system of aerodynamic dipoles radiating a pressure field of amplitude

$$\sum_n \frac{\kappa \cos\theta}{2\pi N|\mathbf{x}|} p_h(\mathbf{y}_n). \tag{31}$$

The field is now independent of the aperture size, providing again that the inequality (30) holds. In aerodynamic flows $\kappa \sim M/l$, so that the intensity of the distant field scattered from the 'transparent' screen is proportional to

$$(\rho U^3 M^3/l^2|\mathbf{x}|^2 N^2)\cos^2\theta. \tag{32}$$

Both these limiting forms are physically interpretable. A screen of low porosity supports a monopole at each aperture with the strength dependent only on the aperture geometry. On the other hand a screen of high porosity supports dipoles, on the solid sections, the strength of the dipoles being independent of the aperture geometry. At grazing incidence there is obviously no scattered field, the screen being parallel to the velocity in the sound ray.

In the absorbent liner screen $R$ must be minimized for maximum absorption (which appears in this model problem as transmission), so that the inequality (30) is aimed for. The scattered hydrodynamic field is generated in this limit according to (31), so that for this to be minimized at the same time as the reflexion coefficient the hole number density $N$ must be maximized. The most efficient non-scattering porous screen is one with the maximum number of small holes per unit area.

## 3. The acoustic properties of turbulence near a baffled array of pistons

Suppose now that in each aperture of the previous problem there is a rigid piston which can move in the $z$ direction with an average impedance $Z_0$. On the piston

$$-\frac{1}{\pi a^2}\int p\, dS = Z_0 v = \frac{Z_0}{i\omega\rho}\frac{\partial p}{\partial z} = \frac{Z_0 Q}{i\kappa a\rho c\pi a}. \tag{33}$$

The geometry is precisely that of the previous section, so is the source field.

Equations (4) and (7) still describe the field, and (4) states that the pressure on the $n$th piston is

$$p(\mathbf{y}_n) = 2p_i(\mathbf{y}) - \frac{1}{2\pi}\sum_{j\neq n}\frac{\exp\left[i\kappa|\mathbf{y}-\boldsymbol{\xi}_j|\right]}{|\mathbf{y}-\boldsymbol{\xi}_j|}Q(\boldsymbol{\xi}_j) - \frac{1}{2\pi}\frac{\partial p}{\partial z}(\mathbf{y})\int_{S_n}\frac{d^2\boldsymbol{\xi}}{|\mathbf{y}-\boldsymbol{\xi}|} \quad \mathbf{y}\in S_n. \tag{34}$$

This gives a force on the $n$th piston from which the 'mass flow' term $Q$ can be computed via (33).

$$\frac{1}{\pi a^2}\int_{S_n} p\,dS = \frac{-Z_0 Q(\mathbf{y}_n)}{i\kappa a\rho c\pi a} = 2p_i(\mathbf{y}_n) - \frac{1}{2\pi}\sum_{j\neq n}\frac{\exp\left[i\kappa|\mathbf{y}_n-\boldsymbol{\xi}_j|\right]}{|\mathbf{y}_n-\boldsymbol{\xi}_j|}Q(\boldsymbol{\xi}_j) - \frac{8Q(\mathbf{y}_n)}{3a\pi^2}, \tag{35}$$

$$-Q(\mathbf{y}_n)\left\{Z_0 - \frac{8}{3\pi}i\kappa a\rho c\right\}\frac{1}{i\kappa a\rho c\pi a} = 2p_i(\mathbf{y}_n) - \frac{1}{2\pi}\sum_{j\neq n}\frac{\exp\left[i\kappa|\mathbf{y}_n-\boldsymbol{\xi}_j|\right]}{|\mathbf{y}_n-\boldsymbol{\xi}_j|}Q(\boldsymbol{\xi}_j). \tag{36}$$

The summation required in (7) can now be performed precisely as it was for (13) to give

$$-\sum_n Q(\mathbf{y}_n)\exp\left[-i\kappa(y_{n1}-y_{01})\sin\theta\right] = \sum_n\frac{2i\kappa a\rho c\pi a p_i(\mathbf{y}_n)\exp\left[-i\kappa(y_{n1}-y_{01})\sin\theta\right]}{(Z_0-[8/3\pi]i\kappa a\rho c)+(\rho c\pi a^2N/\cos\theta)}, \tag{37}$$

so that

$$\frac{\exp\left[i\kappa|\mathbf{x}-y_0|\right]}{2\pi|\mathbf{x}|}\sum_n\exp\left[-i\kappa(y_{n1}-y_{01})\sin\theta\right]Q(\mathbf{y}_n) = \frac{2\rho c\pi a^2 N p_r(\mathbf{x})_{|\mathbf{x}|\to\infty}}{\{Z_0-(8/3\pi)\,i\kappa a\rho c\}\cos\theta+\rho c\pi a^2N} - \sum_n\frac{\exp[i\kappa|\mathbf{x}-\mathbf{y}_n|]}{4\pi|\mathbf{x}|}\frac{4i\kappa a\rho c\pi a p_h(\mathbf{y}_n)\cos\theta}{[\{Z_0-(8/3\pi)\,i\kappa a\rho c\}\cos\theta+\rho c\pi a^2N]}. \tag{38}$$

Equation (7) can now be expressed in its final form, which is written below in a manner that is again suggestive of the physical origin of the various terms:

$$p(\mathbf{x}) \sim p_i(\mathbf{x}) + Rp_r(\mathbf{x}) - \sum_n(1+R)\,Q(\mathbf{y}_n)\frac{\exp\left[i\kappa|\mathbf{x}-\mathbf{y}_n|\right]}{4\pi|\mathbf{x}-\mathbf{y}_n|} \quad \text{as} \quad |\mathbf{x}|\to\infty. \tag{39}$$

or

$$p(\mathbf{x}) \sim p_i(\mathbf{x}) + Rp_r(\mathbf{x}) - \sum_n i\kappa\cos\theta\,\frac{\exp\left[i\kappa|\mathbf{x}-\mathbf{y}_n|\right]}{2\pi|\mathbf{x}-\mathbf{y}_n|}\frac{1-R}{N}p_h(\mathbf{y}_n) \quad \text{as} \quad |\mathbf{x}|\to\infty. \tag{40}$$

Here $R$ is the surface reflexion coefficient associated with a homogeneous surface of impedance $Z$:

$$Z = \frac{Z_0-(8/3\pi)\,i\kappa a\rho c}{\pi a^2N}, \tag{41}$$

$$R = \frac{Z\cos\theta-\rho c}{Z\cos\theta+\rho c}. \tag{42}$$

$Q$ is given by (39) for the monopole strength at each piston and can be interpreted as follows. The pressure on a rigid baffle is twice the pressure in the free field. This pressure induces a motion of the piston which supplements the local pressure by an amount $(8/3\pi)\, i\rho c\kappa a v$. The pressure on the piston face which drives the piston impedance $Z_0$ is therefore

$$2p_h + (8/3\pi)\, i\kappa a\rho c v = Z_0 v, \tag{43}$$

$$v = \frac{2p_h}{Z_0 - (8/3\pi)\, i\kappa a\rho c}. \tag{44}$$

Each piston moving with this velocity induces a monopole of strength

$$Q = i\kappa a\rho c\pi a v = \frac{i\kappa a\rho c\pi a 2p_h}{Z_0 - (8/3\pi)\, i\kappa a\rho c}. \tag{45}$$

These monopoles radiate and are reflected in the surface with reflexion coefficient $R$ so that the scattered field from the pistons is

$$-\sum_n (1+R)\, \frac{2p_h i\kappa a\rho c\pi a}{Z_0 - (8/3\pi)\, i\kappa a\rho c}\, \frac{\exp[i\kappa|\mathbf{x}-\mathbf{y}_n|]}{4\pi|\mathbf{x}-\mathbf{y}_n|}. \tag{46}$$

This sum is in fact that in (38)–(40) so that the physical origin of the scattered field is very clear.

The effective impedance of the surface is seen to be an average of the surface impedance over an area containing several pistons, assuming the pressure is constant in space.

$$Z = \frac{\int p\, dS}{\int v\, dS} = \frac{\dfrac{1}{\pi a^2}\displaystyle\int_{\text{piston}} \left(Z_0 - \frac{8}{3\pi}\, i\kappa a\rho c\right) v\, dS}{N \displaystyle\int_{\text{piston}} v\, dS} = \frac{Z_0 - \dfrac{8}{3\pi}\, i\kappa a\rho c}{\pi a^2 N}. \tag{47}$$

The reflexion coefficient inevitably tends towards $-1$ at grazing incidence, so that this finite impedance surface is quite different in behaviour from the perforated screen at near grazing angles. No field is scattered at grazing incidence in this case in contrast to the monopole field scattered there by the porous screen.

The scattered field is a system of aerodynamic dipoles with axis normal to the screen, centred at the pistons in the limit

$$\rho c\pi a^2 N \gg |z_0 - (8/3\pi)\, i\kappa\rho c| \cos\theta, \tag{48}$$

when the scattered field amplitude will be

$$\sum_n \frac{\kappa p_n(\mathbf{y}_n)}{\pi N|\mathbf{x}|} \cos\theta. \tag{49}$$

In the other limit,

$$|Z_0 - (8/3\pi)\, i\kappa a\rho c| \cos\theta \gg \rho c\pi a^2 N, \tag{50}$$

the scatterers are a system of aerodynamic monopoles and the scattered field amplitude is

$$\sum_n \frac{3\pi}{8} \frac{a}{|\mathbf{x}|}\, p_h(\mathbf{y}_n). \tag{51}$$

The intensities of the monopole and dipole field will scale as usual on the fourth and sixth power of velocity respectively.

The high frequency monopole limit in both perforated screen and 'pistoned' screen are very similar, with the number of apertures and nature of the piston impedance being completely irrelevant to the field. The low frequency limits when the scattered field is dipole are, however, similar in the two problems only as long as $Z_0 \ll \rho c \pi a^2 N$.

## 4. Conclusions

The long-wavelength acoustic properties of a rigid screen perforated with small holes can be deduced from the known response to an incoming wave field of a single aperture in an infinite plane baffle. Acoustic transmission is the only 'absorption' term, so that the real part of the surface impedance must be independent of screen geometry, or even of the existence of the screen, at a value $\rho c/\cos\theta$. The imaginary part is a mass term. The flux $Q$ through unit screen area is $N$ times $4ap_i$, the flux through an individual aperture, so that the average velocity through the homogeneous screen is $-4ap_i N/i\omega\rho$.

However, the surface pressure on the almost rigid surface is approximately $2p_i$ and the imaginary part of the surface impedance is therefore $-i\rho ck/2aN$. The apparent surface mass per unit area is $\rho/2aN$. This argument is essentially Lord Rayleigh's (1896, vol. II, p. 180); he also points out that the conductance is a minimum for circular apertures. The imaginary part of the impedance is therefore a maximum and could easily be reduced by making the apertures non-circular. This point is emphasized by Lamb (1925), who treats the analogous two-dimensional problem.

The ability of the screen to scatter a small-scale turbulence pressure field into sound can be deduced in a similar manner. A hydrodynamic pressure field $p_h$ drives flow through each aperture to create there, on the 'visible' side, a monopole of strength $-4ap_h$ and a monopole of strength $+4ap_h$ that is only partly heard through the screen. If the screen is transparent to sound, then the two sources annihilate each other and a weaker dipole sound is scattered. The parameter determining the transparency of the screen is $4aN/\kappa\cos\theta$. When this parameter is small the screen is acoustically opaque and only the 'visible' monopole is heard. Sound is scattered very efficiently in this limit, the intensity of the omnidirectional scattered field increasing with the fourth power of the flow velocity. In the other limit, which is valid for larger or more numerous holes, or for lower frequencies or near grazing incidence, the screen is acoustically transparent. The weaker dipole scattered field then increases as the sixth power of the flow velocity with a directional peak normal to the screen surface. The most efficient non-scattering porous screen is one with the maximum number of holes per unit area. It is also likely to be composed of highly non-circular holes, a condition Rayleigh showed necessary for the attainment of maximum acoustic transmission.

The second model problem has very similar general characteristics, the only real difference arising from the fact that sound cannot propagate at grazing incidence if the screen has finite piston impedance. The grazing incidence scattered sound is therefore of dipole type at all conditions.

As an order of magnitude of the maximum level that the scattered field can assume, consider monopole sound scattered from a turbulent boundary layer formed on an absorbent porous surface. The wall pressure $p_h$ is likely to be 0·006 of the mean flow dynamic head $\rho U^2$ (at least it is on a smooth surface) so that the acoustic power scattered from unit surface area of turbulent boundary layer of the order $10^{-6}a^2N\rho U^4c^{-1}$.

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